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HEAT TRANSFER FROM A SPHERICAL SOURCE IN A RAREFIED GAS

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The heat-transfer problem from a sphere in a rarefied gas is solved using a model kinetic equation. Diffuse scattering of gas molecules by the sphere surface with arbitrary energy accomodation is assumed.

A solution of the heat-transfer problem from a sphere in a rarefied gas with arbitrary values of the Knudsen number Kn was published in [1, 2]. Lees [1] used his own four-moment method for solution of the Boltzmann equation. In [2] the BGK model equation was used and complete energy accomodation of gas molecules on the sphere surface was assumed.

In the present study an approximate equation of higher order [3] will be used, ensuring a correct value for the Prandtl number. For the boundary condition we will assume arbitrary molecular energy accomodation on the sphere surface. Such a solution is of interest, first, so that model equations may be compared, second, to evaluate the accuracy of the four-moment method, and third, to provide a quantitative estimate of the energy accomodation coefficient for various gases when theory and experiment are compared.

We will consider a sphere of radius R_0, the temperature of which, T_s, differs from the gas temperature T_∞ in the undisturbed region, while $\tau_s = (T_s - T_\infty)/T_\infty \ll 1$. Then the state of the gas is described by a distribution function close to Maxwellian:

$$f(\mathbf{r}, \mathbf{v}) = f_\infty [1 + h(\mathbf{r}, \mathbf{v})], \ \|h\| < 1,$$
$$f_\infty = n_\infty \left(\frac{m}{2\pi k T_\infty}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT_\infty}\right), \tag{1}$$

where n_∞ is the numerical gas density in the unperturbed region and k is Boltzmann's constant.

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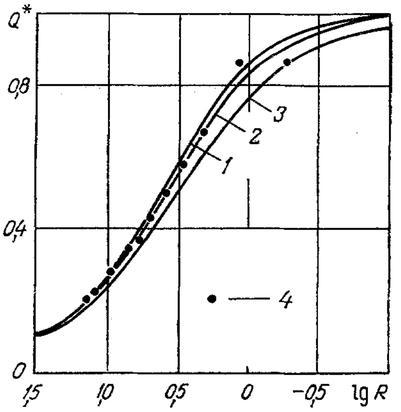

Fig. 1

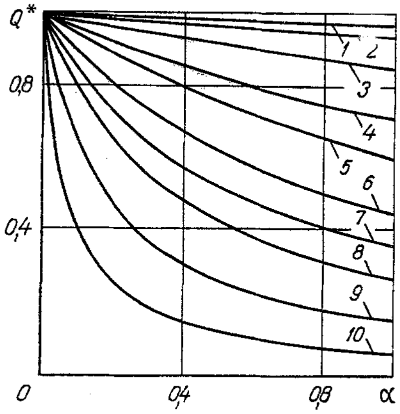

Fig. 2

Fig. 1. Reduced thermal flux Q* versus gas rarefaction parameter R for complete energy accomodation ($\alpha = 1$): 1) present study; 2) BGK [2]; 3) four-moment method [1]; 4) experimental data for air [5].

Fig. 2. Reduced thermal flux Q* versus energy accomodation coefficient α for various R: 1) R = 0.3; 2) 0.5; 3) 1.0; 4) 2.0; 5) 3.0; 6) 5.0; 7) 7.0; 8) 10.0; 9) 20.0; 10) 50.0.

Let the molecules scatter from the spherical surface completely diffusely with a Maxwell distribution function corresponding to a temperature T^+ and density n^+. After linearization, the boundary condition is written in the form

$$f(R_0, \mathbf{c}) = f_\infty \left[1 + \delta + \left(c^2 - \frac{3}{2} \right) \gamma \right], \ (\mathbf{nc}) > 0,$$
$$\mathbf{c} = \left(\frac{m}{2kT_\infty} \right)^{1/2} \mathbf{v}, \ \delta = (n^+ - n_\infty)/n_\infty, \ \gamma = (T^+ - T_\infty)/T_\infty, \tag{2}$$

where $\mathbf{n}$ is the external normal to the boundary surface.

The constants δ and γ are defined by the condition of nonpenetration of the surface and in terms of the energy accomodation coefficient

$$|N^-| = |N^+|, \ \alpha = \frac{|E^-| - |E^+|}{|E^-| - |E_s|}, \tag{3}$$

where N^-, N^+ are the molecular fluxes incident upon and reflected from the spherical surface; E^-, E^+, energy fluxes arriving at and departing from the wall; E_s, departing energy flux when the gas is in thermal equilibrium with the sphere.

Following Eq. (1), the kinetic equation with model collision term from [3] is linearized and written in the following form

$$\mathbf{c} \frac{\partial h}{\partial \mathbf{r}} = \nu + \left(c^2 - \frac{3}{2} \right) \tau + c_r \left(c^2 - \frac{5}{2} \right) Q - h, \tag{4}$$

where

$$\nu(r) = \frac{n - n_\infty}{n_\infty} = \pi^{-3/2} \int \exp(-c^2) h d\mathbf{c};$$
$$\tau(r) = \frac{T - T_\infty}{T_\infty} = \frac{2}{3} \pi^{-3/2} \int \exp(-c^2) \left(c^2 - \frac{3}{2} \right) h d\mathbf{c}; \tag{5}$$
$$Q(r) = \frac{4}{15} \left(\frac{m}{2kT_\infty} \right)^{1/2} \frac{q(r)}{n_\infty k T_\infty} = \frac{4}{15} \pi^{-3/2} \int \exp(-c^2) c_r \left(c^2 - \frac{5}{2} \right) h d\mathbf{c}.$$

Here n and T are the local values of the numerical density and gas temperature; q, radial heat flux density; and the dimensionless radial coordinate r is referenced to the value $2l/\sqrt{\pi}$.

With consideration of boundary condition (2), Eq. (4) is written in integral form, and then with the aid of Eq. (5) is transformed into a closed system of integral moment equations, which can be expressed conveniently in matrix form

$$\mathbf{X} = A\mathbf{X} + \mathbf{B}, \tag{6}$$

where

$$\mathbf{X} = \begin{pmatrix} \nu \\ \sqrt{\frac{3}{2}}\,\tau \\ \frac{\sqrt{15}}{2}Q \end{pmatrix};\quad A\mathbf{X} = \frac{1}{\pi^{3/2}}\int\limits_{V} D\mathbf{X}\,\frac{d\mathbf{r}}{|\mathbf{r}-\mathbf{r}'|^2};$$

$$D = \left(\begin{matrix} I_1 & \sqrt{\frac{2}{3}}\left(I_3-\frac{3}{2}I_1\right) \\ \sqrt{\frac{2}{3}}\left(I_3-\frac{3}{2}I_1\right) & \frac{2}{3}\left(I_5-3I_3+\frac{9}{4}I_1\right) \\ \frac{2}{\sqrt{15}}\left(I_4-\frac{5}{2}I_2\right)(\mathbf{n}\Omega') & \frac{2}{\sqrt{15}}\sqrt{\frac{2}{3}}\left(I_6-4I_4+\frac{15}{4}I_2\right)(\mathbf{n}\Omega') \end{matrix}\right. \rightarrow$$

$$\rightarrow \left.\begin{matrix} \frac{2}{15}\left(I_4-\frac{5}{2}I_2\right)\mathbf{n}\Omega') \\ \frac{2}{\sqrt{15}}\sqrt{\frac{2}{3}}\left(I_6-4I_4+\frac{15}{4}I_2\right)(\mathbf{n}'\Omega') \\ \frac{4}{15}\left(I_7-5I_5+\frac{25}{4}I_3\right)(\mathbf{n}\Omega')(\mathbf{n}'\Omega') \end{matrix}\right); \tag{7}$$

$$\mathbf{B} = \frac{1}{\pi^{3/2}}\int\limits_{\omega_0} d\boldsymbol{\omega} \begin{pmatrix} \delta I_2 + \gamma\left(I_4-\frac{3}{2}I_2\right) \\ \sqrt{\frac{2}{3}}\left[\delta\left(I_4-\frac{3}{2}I_2\right)+\gamma\left(I_6-3I_4+\frac{9}{4}I_2\right)\right] \\ \frac{2}{\sqrt{15}}\left[\delta\left(I_5-\frac{5}{2}I_3\right)+\gamma\left(I_7-4I_5+\frac{15}{4}I_3\right)\right](\mathbf{n}\Omega) \end{pmatrix};$$

$$I_n(s) = \int t^n \exp\left(-t^2-\frac{s}{t}\right)dt;$$

$$\mathbf{n} = \frac{\mathbf{r}}{r};\ \mathbf{n}' = \frac{\mathbf{r}'}{r'};\ \Omega = \frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|};\ \Omega' = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}; \tag{8}$$

$\mathbf{r}_0$ is the radius vector of the sphere surface; the argument of the function I_n in Eq. (7) is $|\mathbf{r} - \mathbf{r}'|$, but in Eq. (8), $|\mathbf{r} - \mathbf{r}_0|$; ω_0 is the solid angle in which the sphere is visible from point $\mathbf{r}$; the integration range V is that part of space, all points of which can be connected to the tip of the radius vector $\mathbf{r}$ by a straight line which does not intersect the sphere.

Using the definitions of the quantities appearing in Eq. (3) from [4], it is simple to obtain a pair of equations of the following form

$$2\sqrt{\pi}R^2\left(\varepsilon+\frac{1}{2}\tau_s\right) = \frac{1}{\pi^{3/2}}\int\limits_{V_0} d\mathbf{r}\int\limits_{\omega_0} d\boldsymbol{\omega}\left[\nu I_2+\tau\left(I_4-\frac{3}{2}I_2\right)\right] - Q\left(I_5-\frac{5}{2}I_3\right)(\mathbf{n}\Omega), \tag{9}$$

$$4\sqrt{\pi}R^2\left[\varepsilon+\frac{\gamma}{1-\alpha}+\frac{1-3\alpha}{2(1-\alpha)}\tau_s\right] = \frac{1}{\pi^{3/2}}\int\limits_{V_0} d\mathbf{r}\int\limits_{\omega_0} d\boldsymbol{\omega}\left[\nu I_4+\tau\left(I_6-\frac{3}{2}I_4\right)\right] - Q\left(I_7-\frac{5}{2}I_5\right)(\mathbf{n}\Omega), \tag{10}$$

where $R = \frac{\sqrt{\pi}}{2}\cdot\frac{R_0}{l} = \frac{\sqrt{\pi}}{2}\mathrm{Kn}^{-1}$; $\varepsilon = \delta - \frac{1}{2}(\tau_s - \gamma)$; V_0 is all the space about the sphere; and the argument of the functions I_n is $|\mathbf{r} - \mathbf{r}_0|$.

The value of the radial thermal flux is determined from the last equation of Eq. (6). In this expression it is necessary to set r = R and integrate over the sphere surface. Then, with consideration of Eqs. (9), (10), we have:

$$Q^* = \frac{Q}{Q_0} = \frac{1}{1-\alpha}\left(1 - \frac{\gamma}{\tau_s}\right), \tag{11}$$

where Q_0 is the free molecular value of Q, equal to

$$Q_0 = \frac{4\alpha}{15\sqrt{\pi}} \frac{R^2}{r^2} \tau_s.$$

Thus, the problem has been reduced to the solution of a system of Fredholm integral equations (6). Since the integrands are symmetric, the variation method of [2] may be used.

We consider a function of the form

$$F(\tilde{\mathbf{X}}, \tilde{\varepsilon}, \tilde{\gamma}) = (E\tilde{\mathbf{X}}, \tilde{\mathbf{X}} - \tilde{A}\mathbf{X} - 2B) + C_{44}\tilde{\varepsilon}^2 + C_{55}\tilde{\gamma}^2 - 2d_4\tilde{\varepsilon} - 2d_5\tilde{\gamma}, \tag{12}$$

where

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix};$$

and the scalar product of two arbitrary functions f and g is determined by the formula

$$(\mathbf{f}, \mathbf{g}) = \int\limits_{V_0} \mathbf{fgdr};$$

the coefficients C_{44}, C_{55}, d_4, d_5 are chosen to make the function stationary for independent variation of $\tilde{X}$, $\tilde{\varepsilon}$, $\tilde{\gamma}$:

$$C_{44} = 2\sqrt{\pi}R^2,\ C_{55} = \frac{4\sqrt{\pi}R^2}{1-\alpha},\ d_4 = -\sqrt{\pi}R^2\tau_s,$$
$$d_5 = 4\sqrt{\pi}R^2\frac{\alpha\tau_s}{1-\alpha}. \tag{13}$$

Equation (12) takes on a stationary value when $\tilde{X}$, $\tilde{\varepsilon}$, $\tilde{\gamma}$ satisfy Eqs. (6), (9), (10), i.e.,

$$\operatorname{stat} F(\tilde{\mathbf{X}}, \tilde{\varepsilon}, \tilde{\gamma}) = F(\mathbf{X}, \varepsilon, \gamma) = -\sqrt{\pi}R^2\tau_s^2\left(\frac{1}{2} + \frac{4\alpha}{1-\alpha}\frac{\gamma}{\tau_s}\right).$$

Comparison with Eq. (11) gives

$$Q^* = \frac{1+7\alpha}{8\alpha(1-\alpha)} + \frac{\operatorname{stat} F}{4\sqrt{\pi}R^2\alpha\tau_s^2}. \tag{14}$$

Following [2], test functions are chosen from the condition of accurate asymptotic behavior of the corresponding macroscopic quantities in the Navier–Stokes region

$$\tilde{\mathbf{X}} = \frac{1}{r}\begin{Bmatrix} X_1 \\ \sqrt{\frac{3}{2}}X_2 \\ \frac{\sqrt{15}}{2}\frac{X_3}{r} \end{Bmatrix},\ \tilde{\varepsilon} = X_4,\ \tilde{\gamma} = X_5. \tag{15}$$

Substituting Eq. (15) in Eq. (12), we obtain

$$F = \sum_{i=1}^{5}\left(C_{ii}X_i^2 - 2d_iX_i + 2\sum_{k>i}^{5}C_{ik}X_iX_k\right). \tag{16}$$

The coefficients C_{ij}, d_i are cumbersome in form and will not be present here.

The constants X_i are defined to make Eq. (16) stationary:

$$\frac{\partial F}{\partial X_i} = 0;\ i = 1, 2, 3, 4, 5. \tag{17}$$

The formulas obtained above can be used for calculations only when $\alpha \neq 1$. The case of complete energy accomodation ($\alpha = 1$) requires individual examination. To do this, it is sufficient to set $\gamma = \tau_s$, while to solve system (6) together with the nonpenetration condition (9), Eq. (12) may be used with $C_{55} = d_5 = 0$. The remaining treatment remains the same as before.

Asymptotic analysis of Eq. (14) as $R \to 0$ or $R \gg 1$ gives the free molecular and continual values of Q*, equal to

$$Q^*(R \to 0) \to 1,\ Q^*(R \gg 1,\ \alpha) = \frac{15\sqrt{\pi}}{8R}\left[\alpha\left(1 - \frac{3.760}{R}\right) + \frac{3.323}{R}\right]^{-1}, \qquad (18)$$

respectively, which coincides with the results of [1, 2] for $\alpha = 1$.

In the intermediate regime Q* is defined numerically to an accuracy of not less than 0.2%.

Figure 1 presents a comparison of the results of the present study at $\alpha = 1$ (curve 1) with those of [1] (curve 3) and [2] (curve 2), and also with experimenta; data for air [5]. The discrepancy between the results obtained with the S-model kinetic equation and the BGK model of [2] does not exceed ~3%. Such agreement indicates the satisfactory agreement of the model equations. However, it should be noted that such agreement occurs only with a special choice of the collision parameter in the BGK model. The collision frequency must be selected such that the model properly describes thermal flux relaxation in the gas. It is obvious that this approach is impossible in description of phenomena produced by both the thermal conductivity and viscosity of the gas simultaneously. In such problems the S-model kinetic equation is to be preferred.

Comparison of the present results with the four-moment method of [1] shows their complete agreement in the continual and free-molecular heat transfer regimes. However, in the intermediate region their disagreement reaches ~12%, due to the approximations of the four-moment method.

Comparison of the theory for $\alpha = 1$ with experimental data for air [5] indicates satisfactory agreement for $R \geqslant 2$. Unfortunately, the experimental technique used did not permit measurements at low pressure, since the effect of the external volume became appreciable and the experimental points showed wide scattering.

Figure 2 shows the dependence of the dimensionless quantity Q* upon energy accomodation coefficient α for various values of the gas rarefaction parameter R. With decrease in α toward zero Q* increases to unity independent of R, but since then $Q_0 \to 0$, the thermal flux $Q \to 0$, which is an obvious result for an absolutely nonaccomodating sphere. It is also evident from Fig. 2 that Q* depends weakly on α at $R \leqslant 0.5$. This means that in the range $R \leqslant 0.5$ the thermal flux Q can be considered approximately proportional to α.

In conclusion, we note that comparison of the solution obtained with reliable experimental data could provide information on energy accomodation coefficients for gas-surface interactions.

NOTATION

R_0, sphere radius; n, numerical density of gas molecules; T, gas temperature; α, energy accomodation coefficient; r, dimensionless radial coordinate; l, mean molecular free path length; $R = \sqrt{\pi} R_0 / 2l$, gas rarefaction parameter; m, molecular mass; **v**, molecular velocity; q, thermal flux density.

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